① HFS in Na

a) 2 HFS levels

b) $\vec{F} = \vec{I} + \vec{j}$ $\quad I = \frac{3}{2}$ $\quad j = \frac{1}{2} \Rightarrow$ $F = 1$ or $F = 2$

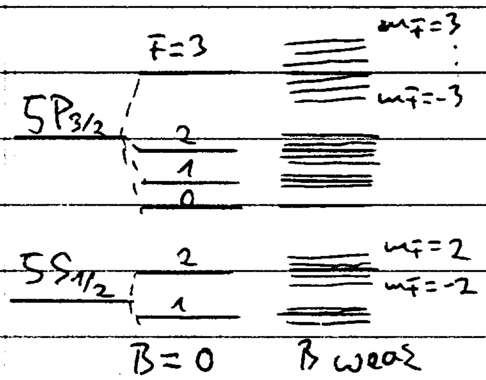
c) $F = 7/2$ or $F = 9/2$

d) $\Delta E = 1770 \cdot \frac{F_>(^{24}Na)}{F_>(^{23}Na)} \cdot \frac{g_I(^{24}Na)}{g_I(^{23}Na)} = 1770 \cdot \frac{9/2}{2} \cdot \frac{0.423}{[2.2161/(\frac{3}{2})]}$

$$\Delta E = 1770 \cdot \frac{9}{4} \cdot 2{,}863 = 11402 \text{ MHz}$$

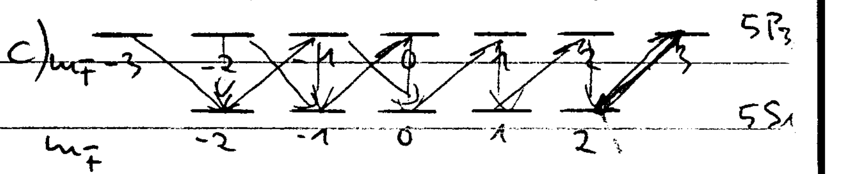
②

a)



b) to keep the balance between B field and decreasing Doppler-shift of the slowing atoms.

c)



once the atoms reach $5P_{3/2}$ $m_F = 3$, they can only fall back to $5S_{1/2}$ $m_F = 2$ -> 2 level system

d) $\Delta E = |(E_{HFS})_P - (E_{HFS})_S|$

$= |(g_F m_F \mu_B B)_P - (g_F m_F \mu_B B)_S|$

$= \mu_B B$

e) $p = \frac{h}{\lambda} = m\Delta v \Rightarrow \Delta v = \frac{h}{\lambda m} \approx 0.006 \frac{m}{s}$

$\frac{500\frac{m}{s} - 200\frac{m}{s}}{0.006\frac{m}{s}} = 50.000$ absorptions

f) $v = \frac{s}{t} \Rightarrow s = vt = \frac{300\frac{m}{s}}{2} \cdot 50000 \cdot 26\mu s$

$= 0.195$ m

g) $x = v_0 t - \frac{a}{2}t^2$ $\quad a = \frac{\Delta v}{\Delta t} = \frac{0.006\frac{m}{s}}{26\mu s}$

$v = v_0 - at$

$\Rightarrow v = (v_0^2 - 2xa)^{1/2}$

Zeeman splitting has to compensate Doppler-shift:

$\mu_B B = h \cdot \nu_L \frac{v}{c} \Rightarrow B = \frac{h}{\mu_B} \frac{v}{\lambda_L}$

$B = \frac{h}{\mu_B \lambda_L} \sqrt{v_0^2 - 2xa}$

h) the 2 level system becomes:

$m_F = -2 \leftrightarrow m_F = -3$

For σ^+ polarized light, the atoms with $v \approx 0$ are in resonance with the laser -> they might be pushed back.

For σ^- light the laser has to be detuned -> no problem occurs.

(3) a) $E(2s) = -13.6\,\text{eV} \cdot \frac{1}{(n-0.4115)^2} = -5.39\,\text{eV}$

$E(2p) = -3.54\,\text{eV}$ $\quad\Rightarrow\quad \Delta E(2s \to 2p) = 1.85\,\text{eV}$

$E(3p) = -1.55\,\text{eV}$ $\quad\quad\quad \Delta E(2s \to 3p) = 3.84\,\text{eV}$

b) $E = -13.6\,\text{eV} \cdot Z_{eff}^2 \cdot \frac{1}{n^2} = -3.54\,\text{eV} \Rightarrow Z_{eff} \approx 1.02$ (2p)

$Z_{eff} \approx 1$ (3p)

c) For excited electrons the remaining electrons screen the nucleus more effective.

d) $(1s2p) \to {}^1P_0, {}^3P_0, {}^3P_1, {}^3P_2$

e) ${}^3P_0 \quad {}^3P_1 \quad {}^3P \quad\quad {}^1P_0 \quad \to E$

f) $(2p^2) \to {}^1S_0, {}^1D_2, {}^3P_0, {}^3P_1, {}^3P_2$

${}^3P_0 \quad {}^3P_1 \quad {}^3P_2 \quad {}^1D_2 \quad {}^1S_0 \quad \to$

(4) a) $H\psi = -\frac{\hbar^2}{2m}\psi'' = E\psi \Rightarrow \psi = A\cos\alpha x \quad , B\sin\beta x$

boundary condition: $\psi = 0$ for $x = \pm a$

$\Rightarrow \cos\alpha a = 0 \Rightarrow \alpha = \frac{\pi}{2a} \qquad \sin\beta a = 0 \Rightarrow \beta = \frac{\pi}{a}$

Normalisation: $A^2\int_{-a}^{a}\cos^2\frac{\pi}{2a} = a A^2 \stackrel{!}{=} 1 \qquad B^2\int_{-a}^{a}\sin^2\frac{\pi}{a} = a B^2 \stackrel{!}{=} 1$

$\Rightarrow A = B = \frac{1}{\sqrt{a}}$

$\Rightarrow \psi_0 = \frac{1}{\sqrt{a}}\cos\frac{\pi}{2a}x \qquad \psi_1 = \frac{1}{\sqrt{a}}\sin\frac{\pi}{a}x$

b) $E_0 = \frac{\hbar^2}{2m}\left(\frac{\pi}{2a}\right)^2 \qquad E_1 = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2$

c) $V(x) = V(-x) \Rightarrow \langle\psi|V|\psi\rangle \neq 0$!

d) $\Delta E_0 = \frac{\delta}{a}\int_{-a/3}^{a/3}\cos^2\frac{\pi}{2a}x\,dx \qquad \cos^2 x = \frac{1}{2}(1+\cos 2x)$

$= \frac{\delta}{a}\int\frac{1}{2}\left(1+\cos\frac{\pi}{a}x\right)dx = \frac{\delta}{2a}\left(x + \frac{a}{\pi}\sin\frac{\pi}{a}x\right)\Big|_{-\frac{a}{3}}^{a/3}$

$\Delta E_0 = \delta\left(\frac{1}{3} + \frac{\sqrt{3}}{2\pi}\right) = 0.61\,\delta$

$\Delta E_1 = \frac{\delta}{a}\int_{-a/3}^{a/3}\sin^2\frac{\pi}{a}x\,dx \qquad \sin^2 x = \frac{1}{2}(1-\cos 2x$

$= \frac{\delta}{2a}\int\left(1-\cos\frac{2\pi}{a}x\right)dx = \frac{\delta}{2a}\left(x - \frac{a}{2\pi}\sin\frac{2\pi}{a}x\right)\Big|_{-a/3}^{a/3}$

$\Delta E_1 = \delta\left(\frac{1}{3} - \frac{\sqrt{3}}{4\pi}\right) = 0.195\,\delta$